Dynamical Markov States and the Quantum Core

Some new mathematics for 21st Century science

An illustrated essay based on a talk given at the 20th annual meeting of the Society for Scientific Exploration June 7, 2001

By Tom Etter

Boundary Institute

P.O. Box 3580, Los Altos. CA 94024

etter@boundaryinstitute.org

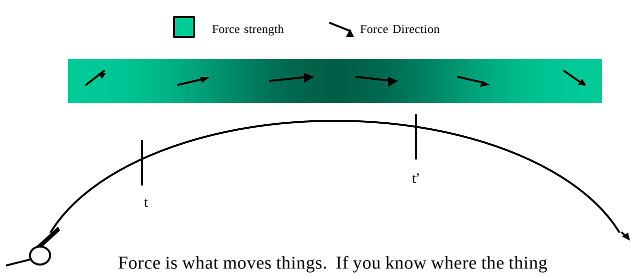
The Birth of the Restless Universe

Physics as we know it was born in around 1600 when ***velocity*** was added to the concept of state. Though this happened with very little fanfare, it was a very big change, since it implies that the *present moment* by its very nature contains something of the *past* and the *future.*

Aristotle and his followers believed that motion must always be sustained by force. A typical Aristotelian object is a beached sailboat. To move it you must haul it, and the moment you stop hauling it, it stops moving. A sailboat on the water will stop shortly after the wind stops, but not *instantly,* which brings up the problem of *momentum.* The momentum problem is much worse in the case of a cannon shot, where, after the initial quick push by the exploding powder, some kind of mysterious hidden force must take over the whole job of moving the cannon ball to its destination.

Aristotelian Cannon Ball

Determinism Involves Two
Kinematic States



Force is what moves things. If you know where the thing is at t, and you know the forces pulling on it, then you know where it is at t'. Think of hauling a beached boat.

It takes an invisible force that changes in complicated ways to move the Aristotelian cannon ball to its destination.

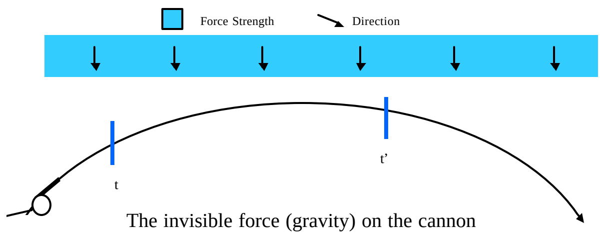
The Birth of the Restless Universe, 2

As we know, Galileo and Newton solved the momentum problem by having force act on *velocity* rather than on *position*. It's very important to realize that this solution was not just the correction of certain mistaken beliefs but *a redefinition of the very concepts of state and force*. Aristotelian physics can always explain the observed lawfulness of the appearances by invoking invisible forces. Newton, too, postulated an invisible force called gravity to explain planetary motion; in that respect he did not go beyond Aristotle. Where his system differed from Aristotle's was in its profound simplicity, unity and explanatory power..

There is a lesson here that is of more than academic interest, since today's science has in certain important and regrettable ways regressed to Aristotelian thinking.

Newtonian Cannon Ball

Determinism involves two dynamical States



The invisible force (gravity) on the cannon ball is *constant* and acts on *velocity.*

The *state* of the cannon ball includes both position and velocity. The state at t determines the state at t' and vice versa..

Kinematic and dynamical states

A *kinematic state* is a state which can be specified without mentioning *change.* Examples:

- The position of a moving body
- The state of a chessboard
- The state of a computer
- The state of a statistical ensemble

A *dynamical state* is a state which *incorporates some aspect of how it is changing*. A Newtonian state is dynamical, since one of its components is the rate at which the other is changing. Quantum states, as currently conceived, are not dynamical .

Why This Matters Today

A second revolution in physics occurred in the latter part of the 19th century when physicists started applying the theory of probability to swarms of atoms. This led to precise explanations of many basic phenomena such as diffusion and mixing processes, chemical reactions, and thermodynamic equilibrium. These are subsumed under the broader category of *Markov process,* which also encompasses computers and other information processing devices.

Obviously this revolution was a great advance, and without it we'd still be riding horses and dying young of today's easily curable diseases. In another sense, however, it was a step backwards. The *state* of a swarm of atoms, or of a puff of smoke, or of a computer, is, like the state of a beached sailboat, a *perfectly Aristotelian concept*. Gone is the Newtonian presence of past and future in these states, which are now just *slices* of history, like cards in a deck. In the terminology of this presentation, the states of a Markov process are *kinematic.*

What We Can Do About It Today

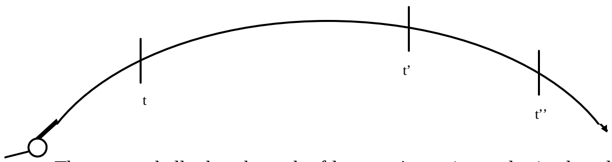
The good news is that they don't *have* to be kinematic. As we'll see, there is a well-defined concept of dynamical state that can be used to describe any Markov process. We can always revert to a kinematic descriptions, and there are indeed many "beached sailboats" in information science, but in certain cases of great interest we find that:

Dynamical states make things very much simpler.

They also offer a hint of larger vision, which is the explanation of space, time and matter as special configurations within a more fundamental domain of being. Today we'll get a glimpse of this promise by seeing how Markovian dynamics contains the logical core of quantum mechanics.

Least Action

The key mathematical idea needed to bring the Newtonian revolution to probability theory and information science was anticipated by the French priest Maupertuis in around 1730 when he proposed a new foundation for mechanics based on *finality.* It was later shown that his mechanics, despite its very different formulation, is mathematically equivalent to Newton's.



The cannon ball takes the path of least *action* to its predestined goal.

Unlike Newtonian determinism, least action is a *three-state kinematic law*. The two states at times t and t'' determine the state at the intermediate time t'. Notice that these three states are not Newtonian but *Aristotelian*.

Three-way kinematics

Least action is not only a three-state law but a *three way* law, in the sense that not only do the first and third states determine the second, but the first and second states determine the third, and the second and third states determine the first

Here are two other familiar three-way laws:

Boyles law: PV = kT

The logical behavior of the XOR gate, where x = (y XOR z) and y = (x XOR z) and z = (x XOR y)

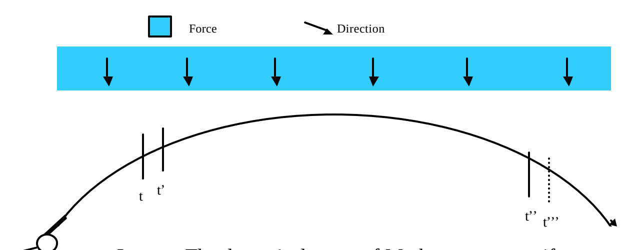
Approximately Newtonian Cannon Ball

Because of the uncertainty principle in quantum mechanics, instantaneous velocity can no longer be regarded as a viable physical concept. Newtonian dynamics is at best an approximate theory.

This requires that we think of dynamical states as having a certain *duration* t'-t. W can redefine the approximate Newtonian dynamical state at t as the *pair* of kinematic states at t and t'. The dynamics of these approximate states is not approximate, however, but is a precise consequence of the three-way kinematic law.

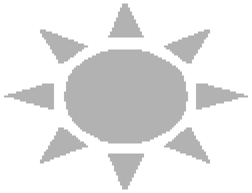
Newton Revisited

By the three way law the states at t and t' determine the states at t'' and t'''



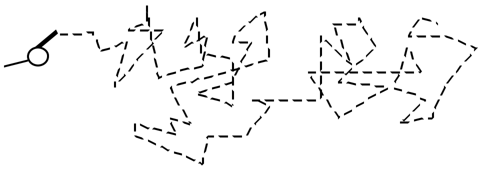
Lesson: The dynamical states of Markov processes, if they exist, will probably have duration. We should thus first look for a three-way Markovian kinematics.

Three Ways to Look at a Puff of Smoke



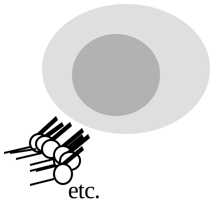
Particles of Smoke

Consider a very very small cannon that shoots a particle of smoke.



The initial velocity of this tiny projectile will be almost completely forgotten after it has been randomly bumped by a few dozen aggressive mole cules, so its position at t is the only aspect of its state that bears on its probable whereabouts at t'. For the Greeks, such random wandering would have meant the end of law and the triumph of chaos. But in our more chaotic modern world, it is the beginning of a new conception of law called *statistical determinism*.

Puffs of Smoke



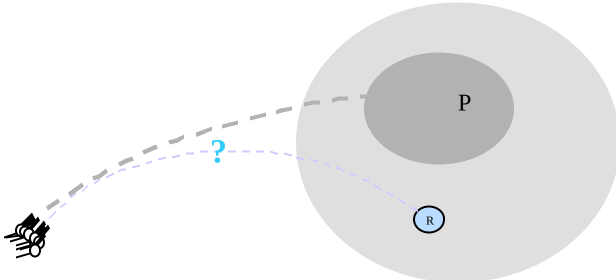
Replace our tiny cannon by a billion billion replicas

The state of a puff of smoke is always *kinematic.*

Smoke, like a beached boat, doesn't have *momentum.*

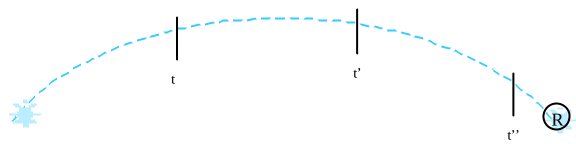
Or does it?

Drifting Smoke



Within the halo of smoke that has randomly drifted away from the main puff, choose a small region R. We'll now pay attention to the smoke particles in R alone. *By what path did this sub-puff get to R?*

How does the blue sub-puff move?

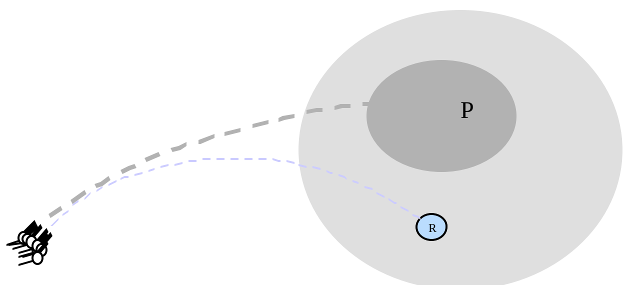


It moves like a cannon ball! That's because it gets to R by the path of *least information* and it can be shown that these paths satisfy Newton's laws, as reformulated by Maupertuis and Lagrange[1]

Least Miraculous Action

Suppose the other smoke particles were retroactively annihilated. Then the behavior of our blue puff would be miraculous. But the miracle that produced it would not be capricious or extravagant; on the contrary, it would be the most *miserly* miracle that the theory of probability allows, in that each *step* would depart from the *expected* step in such a way that the sum total strain on our credulity, however great, is as small as possible. That, in informational terms, is least action.

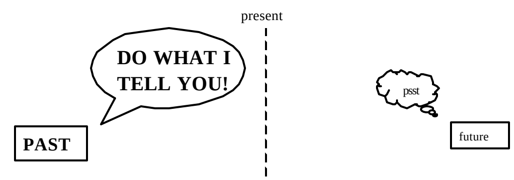
How to put a constraint on the future



Just ignore all the smoke that doesn't end up in R

Backwards time in the real world

Some constraints are stronger than others



Times arrow is associated with the second law of thermodynamics, which says that entropy never decreases. When the constraint on the future is very small compared to that on the past, as is certainly the case in the everyday world, then the second law still holds. Entropy, however, is now the sum of *two parts*, one increasing *forward* in time and the other increasing *backward* in time.[1] The former manifests itself in what we think of as ordinary events, the latter in "miraculous" events.

The lesson of this example

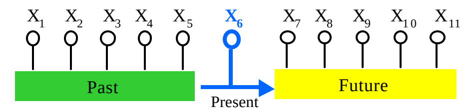
The motion of the blue puff of smoke illustrates a *Markov process* whose orderly behavior is revealed by a three-state kinematic law.

This raises the question of whether three-state kinematic laws apply to other kinds of Markov processes.

We shall now see that they do.

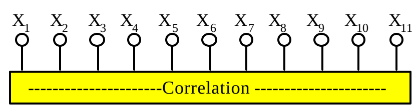
Markov Processes

The concept of a Markov process came out of trying to formalize our intuitive belief that information from the past can only reach the future via the present.

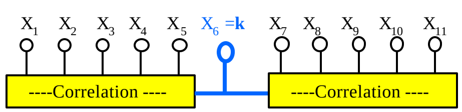


A Markov process, as usually defined, is a succession of state variables in which the probability of a future event, given both the past and the present, is equal to the probability of that event given the present alone. This definition introduces a spurious asymmetry between past and future, however, so we'll next look at a mathematically equivalent definition whose wording is symmetrical in past and future.

Markov Processes: A Better Definition



A Markov process is a succession of correlated random variables such that assigning a fixed value to any one of them makes those before independent of those after[2].

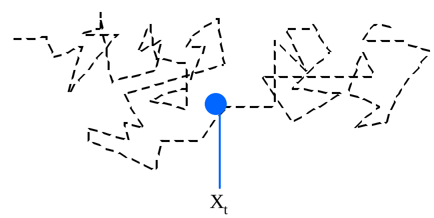


Fixing X_6 by assigning it the value k makes X_1 … X_5 independent of X_7 … X_{11}

Notice that the above sequence in reverse is still a Markov process.

A Simple Example

Given the position X_t of the smoke particle at time t, the wanderings of that particle before t are independent of its wanderings after t.



The Kinematic State of a Markov process

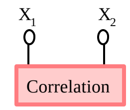
By the *state* of a Markov process we shall mean the *probability distribution* on a state variable rather the actual value of that variable. We are of course now referring to a *kinematic* state; the dynamical state is still to come.

The state of a single smoke particle is its probability distribution over space.

The state of a puff of smoke, on the other hand, is the probability distribution over *all possible spatial distributions* of the smoke particles. The vast majority of these are very nearly of the same shape, so the actual shape is very predictable This is the law of large numbers.

Both the particle and the puff are Markov processes.

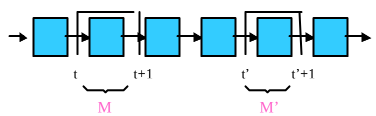
Digram Matrices



To describe two correlated variables you must specify the probability of each pair of their values. It's convenient to arrange these *digram* probabilities in a matrix that we'll call a *digram matrix*.

As we'll soon see, digram matrices can be used in place of transition matrices as the *transformations matrices* on the Markov state. This is how we'll get to the "Newtonian" way of looking at Markov processes, and then onward to quantum mechanics.

Digram States in Markov Processes



The digram matrix on the pair of variables at t and t+1 in a Markov process will be called the digram *state* at t.

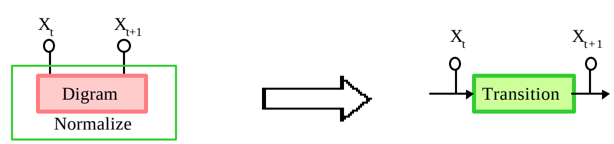
It can be shown that the correlation among all the variables in a Markov process is determined if we know its digram states. To put it another way, a Markov process is a process that is *composed* of its digram states.

Might it be that digram states are governed by simple laws, like Newton's laws, that capture a new fundamental order? Are digram states the Newtonian states of information science?

Transition Matrices

A *transition probability* is the probability that a certain value of the state variable X_t will lead to a certain value of the state variable X_{t+1}. Like digram probabilities, transition probabilities can conveniently be arranged in a matrix, which we'll call a *transition matrix*.

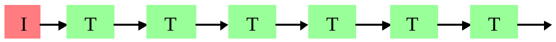
We can calculate the transition matrix from X_t to X_{t+1} by dividing each digram probability by the sum of its column.



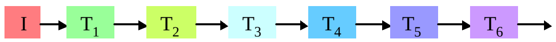
Theorem: The transition matrix at t applied to the state (vector) at t gives the state (vector) at t+1.

Homogeneous and Heterogeneous

Transition probabilities are the analogues of forces. Every Markov process can be represented by a succession of transition matrices applied to an initial state.



Markov processes that result from applying a *constant* transition matrix to an initial state are called *homogeneous*. Think of a homogeneous Markov process as a beached boat being dragged by a steady workhorse. Another name for a homogeneous Markov process is a *Markov chain*.



A Markov process whose transition matrix varies is called *heterogeneous*; think of it as a beached boat being hauled by a party of revelers.

Time Reversal Revisited

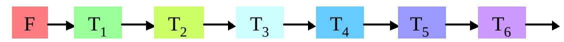
The entropy of the state of a (non-equilibrium) homogeneous Markov process always increases with time.

The time-reversal of a Markov process is a Markov process.

Is the time-reversal of a homogeneous Markov process a homogeneous Markov process?

No, it can't be. Why not? Because its entropy decreases with time!

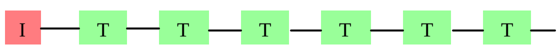
Does time reversal make Markov processes unlawful?



The transition matrix representation of a reversed homogeneous process, where F is the final state of the forward process.

Digram Boxes to the Rescue

Instead of *applying* a series of *transition matrices* to an initial state, we can get the same Markov process by *linking*[3] a series of *digram matrices* to an initial state. We'll refer to these matrices as *digram transformation boxes* to distinguish them from digram states.

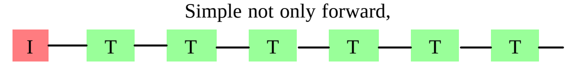


Digram box linking, which is based on the mathematics of *relations* rather than of *functions*, is a more general operation than the composition of transition matrices.. For instance, we can link a state vector to the end of a Markov process as a *second* boundary condition on the process, something that literally makes no sense if we use transition matrices. By so doing, we accomplish in a general way what singling out the "final" blue puff did in the smoke puff example. We have, in effect, created a general *three-state law*.

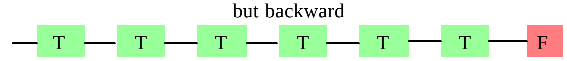
It's important to keep in mind that we are still dealing with Markov processes. It's still the same old cannon ball, but we have found a new viewpoint from which it looks much simpler - indeed, it looks *homogeneous*.

How Things Got Simpler

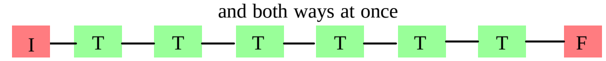
Simple not only forward,



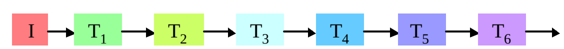
but backward



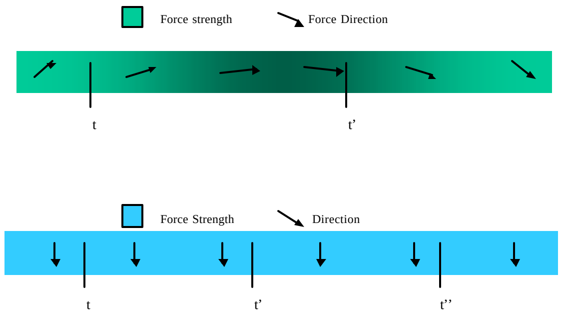
and both ways at once



Instead of this



Reminder: Aristotelian and Least-action Cannon Balls



Three Aristotelian states do a better job of making sense out of cannon ball motion than two.

The New Mathematical Discovery

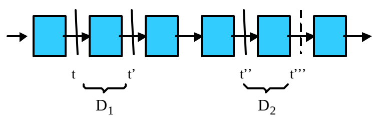
The least-action principle for Markov processes is equivalent to a “Newtonian” analysis based on *dynamical* states

Reminder: Kinematic and Dynamical states

A *kinematic state* is a state that can be completely defined at a single time and thus in itself is independent of change.

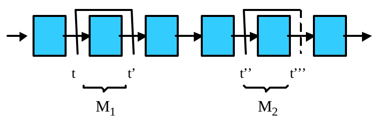
A *dynamical state* is a state that has *duration* and thus incorporates some aspect of how it is changing.

Dynamical determinism



Follows from least information. The “Newtonian” Markov states *must* have duration; they don’t exist in the limit (generalized uncertainty principle)

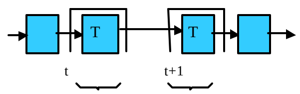
Digram State



Generalizes Newtonian dynamical state. Given by a *matrix* at a pair of times. Its *duration* involves a richer binding of past and future.

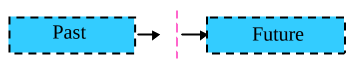
Digram states reveal *uniformity* in a broader class of Markov processes than Newtonian dynamical states do

Digram state dynamics



M_t $M_{t+1} = T \circ (T^{-1}(T^* \circ M_t)T)$

And now for something completely different



L_t

Link State defined

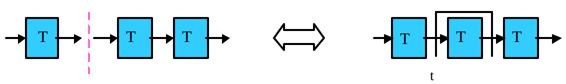
Separate the future from the past and take the matrix of probabilities on the two broken ends

Link states are neither kinematic nor dynamical. They are something else. They are also very powerful tools.

Link states are inter-definable with digram states

Link state at t

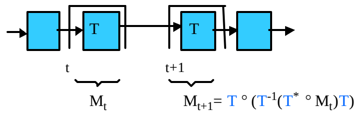
Digram state at t



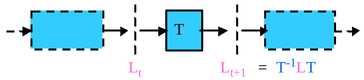
$L_t = T(T^* \circ M_t)$

$M_t = T \circ (T^{-1} L_t)$

Digram state dynamics (observable)



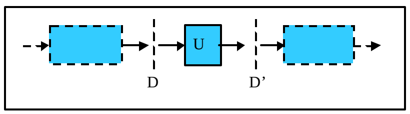
Link state dynamics (unobservable)



Quantum mechanics

When constraints on past and future are equally balanced *at all times*, dynamical Markov theory reduces to quantum mechanics.

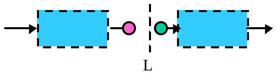
The quantum core



Informally, the quantum core is quantum mechanics stripped of all references to space, time and matter. We can think of it as just that part of quantum physics needed for the logical design of quantum computers. Formally, the quantum core is given by von Neumann's two fundamental laws governing quantum change and measured probability, namely $D' = T^{-1}DT$ and probability(P) = trace(PD).

If we interpret the *link state matrix* as the *density matrix* and require that it always have perfect past-future symmetry (that is, row-column symmetry), then *digram state dynamics turns into the quantum core.*

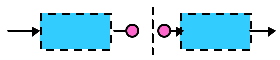
The wave function



The wave function only applies to *pure states*, which are given by link states matrices which are *separable* into pairs of vectors.

The general pure link state has two *different* vectors.

The quantum wave function

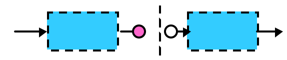


In the quantum case these two vectors are equal, so we need specify only one.

A kinematic state is specified by one vector.

Our worst mistake in trying to understand quantum mechanics has been to confuse the *wave state* at t with a *kinematic state* at t.

The classical "wave function"



In the classical case the second of these two vectors is always "white" (all probabilities equal), so we only need specify the first.

The Quantum Booby Trap

- Pure states need two vectors
- Quantum pure states are degenerate
  - Both vectors are the same
- Classical pure states are degenerate
  - One vector is always white

It is thus natural to confuse pure quantum states with classical kinematic states. The quantum square law for probabilities was a clear warning that this is wrong, but it required link theory to clear up the confusion.

The Realm of Genuine Quantum Mysteries

- Compound identity
- Relative AND
- Intentionality as subject-object polarity
- Dynamical Markov states

The reward for solving them:

A genuine theory of matter and mind