Discrete Motion and the Emergence of Space and Time

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Abstract: *In this paper we give a simple and primitive definition of discrete motion beginning prior to the usual notions of space and time. We show how velocity, the relativistic addition of velocities, and the Lorentz factor naturally emerge from simply counting steps in a sequence of discrete motions.*

Time is nature's way of keeping everything from happening at once. Space is what prevents everything from happening to me. -- (attributed to) John Archibald Wheeler, physicist

Introduction

In this informal paper, we explore the simplest possible definition of motion, namely a change in position. There is no assumption of pre-existing space or time, and these are seen to emerge naturally from simple considerations. Natural consequences include a maximum velocity, and velocity addition that is consistent with Special Relativity.

Consider a simple discrete motion in one dimension consisting of a sequence of *steps* to the left or the right along a line. Steps take place in one direction or the other, and have no size. There is no clock present, and so an event occurs (time "passes") only when there is a step.

A typical sequence of steps might look like this:

$$+ \quad + \quad - \quad + \quad - \quad + \quad + \quad + \qquad\qquad (S_1)$$

that is, 6 steps to the right and 2 steps to the left, for a net progress of 6 - 2 = +4 to the right in a total of 8 steps.

Velocity

It is natural to define a discrete *velocity* just by *counting*, as

$$v = \frac{netsteps}{totalsteps} = \frac{n^+ - n^-}{n^+ + n^-}$$

where n^+ is the number of + steps and n^- is the number of − steps. The velocity in this example is thus

$$v = \frac{6-2}{6+2} = \frac{1}{2}$$

The sequence

$$+ \quad + \quad + \quad + \quad - \quad + \quad + \quad + \qquad\qquad (S_2)$$

is another example, in this case with velocity v = (7-1)/(7+1) = 3/4. Again the total number of steps plays the role of time in our definition of velocity.

Note that velocity is independent of the order of the + and − steps, and may be thought of as a fraction of the ultimate speed c = 1, a string of all + steps (or c = -1, being a string of all − steps). We need not be concerned with the size of the steps in either spatial or temporal terms -- they are merely indivisible units. Thus c is thus the natural maximum velocity, being one step in "space" for each and every step in "time".

Addition of velocities

Imagine now a sum of two independent motions defined as above, with one displacement relative to the other. Obviously, there are 4 possibilities for the combined motions: both +, both −, or opposing motion (+− or −+). This can be illustrated by combining (adding) the two particular sequences above:

S_1:	+	+	−	+	−	+	+	+
S_2:	+	+	+	+	−	+	+	+
Sum:	+	+		+	−	+	+	+ .

Thus the result string contains six + and one − motions. In the one case of opposing components in this example, the result is 0, no motion, no event at all. For this example, the velocity of the sum string is then (6-1)/8 = 5/8. However, a viewer of the sum string will only see 7 total steps, not 8, since +− and −+ motions cancel, see below.

We are only interested in the average velocities, so we will next consider addition of velocities in the general case using a normalized statistical argument.

The general case

In general, these strings represent sequences of independent events that have known distributions (number of + and − overall), but which are not in any particular order nor correlated with each other. We will take the probability of a move to the right or to the left as

$$p_1^+ = n_1^+ / N \quad \text{and} \quad p_1^- = n_1^- / N$$

respectively, where

$$n_1^+ + n_1^- = n_2^+ + n_2^- = N$$

and thus

$$p_1^+ + p_1^- = p_2^+ + p_2^- = 1 \quad .$$

First consider the classical sum of two strings in this form given by

$$v_1 + v_2 = \frac{p_1^+ - p_1^-}{p_1^+ + p_1^-} + \frac{p_2^+ - p_2^-}{p_2^+ + p_2^-} = \frac{2(p_1^+ p_2^+ - p_1^- p_2^-)}{p_1^+ p_2^+ + p_1^+ p_2^- + p_1^- p_2^+ + p_1^- p_2^-} \quad .$$

For example, summing the two strings S_1 and S_2 above yields

$$v_1 + v_2 = \frac{2 * \left(\frac{6}{8} * \frac{7}{8} - \frac{2}{8} * \frac{1}{8} \right)}{\frac{6}{8} * \frac{7}{8} + \frac{6}{8} * \frac{1}{8} + \frac{2}{8} * \frac{7}{8} + \frac{2}{8} * \frac{1}{8}} = \frac{5}{4}$$

in agreement with the common-sense classical notion of adding velocities, in this case

$$v_1 + v_2 = \frac{1}{2} + \frac{3}{4} = \frac{5}{4}$$

but which of course exceeds the stated maximum velocity of 1.

Derived Time

Suppose we make a simple change to the above sum expression based on the postulate of total steps as our measure of time: *When the summed motion is zero no time passes*. That is, when there is no net motion, there is no event at all, and our "clock" (the count of total steps) does not increment either. Thus the total number of time steps in the denominator will not include those cases where the two motions are opposite p^+p^- and p^-p^+. In other words, only actual net + or − motions of the result are counted in the total steps, and all motion is the same size -- one step.

Using the derived time in this way, the effective velocity now becomes

$$v_{1+2} = \frac{\cancel{2}(p_1^+ p_2^+ - p_1^- p_2^-)}{p_1^+ p_2^+ + \cancel{p_1^+ p_2^-} + \cancel{p_1^- p_2^+} + p_1^- p_2^-}$$

$$= \frac{p_1^+ p_2^+ - p_1^- p_2^-}{p_1^+ p_2^+ + p_1^- p_2^-}$$

-- just that defined by the cases where motion is present in the sum.

Substituting velocities for probabilities again, a little algebra yields

$$v_{1+2} = \frac{v_1 + v_2}{1 + v_1 v_2}$$

which is of course just the addition law under the Lorentz transform of Special Relativity with maximum speed $c = 1$.

In the above example, the relativistic summed velocity is then

$$v_{1+2} = \frac{\frac{6}{8} * \frac{7}{8} - \frac{2}{8} * \frac{1}{8}}{\frac{6}{8} * \frac{7}{8} + \frac{2}{8} * \frac{1}{8}} = \frac{10}{11}$$

as expected, and does not exceed the maximum velocity of $c = 1$.

Interpretation

Here we see the real nature of relative motion and the basis for the space and time dilation of Relativity. The "passage" of time is derived from the motion itself and, thus our clock doesn't tick when there's no motion. The usual notions of space and time emerge naturally from discrete events. Further work will be necessary to extend this basic approach to all of Special and General Relativity. For more on the origins of space as distinctions, and time as generated by loops in space, see Shoup (1994).

Acknowledgements

To the best of my knowledge, the basic idea presented here was first developed and published by physicist Irving Stein (1996) and later discovered by Thomas Etter (1998) as well. A similar idea has also been explored in a web page by Kevin Brown (n.d.). I am grateful for discussions on this subject with Tom Etter, Andrew Singer, Ken Wharton, and Joseph Depp.

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